lesson 111 practice a lines that intersect circles answers

Lesson 111 Practice: A Lines That Intersect Circles Answers

Understanding the relationship between lines and circles is essential in geometry. In Lesson 111, we delve into the concept of lines that intersect circles, exploring various scenarios and providing solutions to practice problems. This article outlines key concepts, provides examples, and discusses answers to practice exercises related to lines intersecting circles. By the end of this article, readers will have a deeper understanding of the geometric principles at play and will be better prepared for related problems.

Understanding the Basics of Circles and Lines

Before diving into the specifics of lines that intersect circles, it is important to review some fundamental concepts.

The Circle

A circle is defined as a set of points in a plane that are equidistant from a fixed point known as the center. The distance from the center to any point on the circle is known as the radius. The standard equation of a circle in a Cartesian coordinate system can be represented as:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where (h, k) is the center of the circle, and r is the radius.

The Line

A line is defined as a one-dimensional figure that extends infinitely in both directions. The equation of a line in slope-intercept form is given by:

$$ y = mx + b $$

where m is the slope of the line, and b is the y-intercept.

Types of Intersections Between Lines and Circles

When a line intersects a circle, there are three possible scenarios:

1. No Intersection: The line does not touch the circle at any point. This occurs when the distance from the center of the circle to the line is greater than the radius.
2. Tangent Intersection: The line touches the circle at exactly one point. This happens when the distance from the center of the circle to the line is equal to the radius.
3. Secant Intersection: The line intersects the circle at two points. This occurs when the distance from the center of the circle to the line is less than the radius.

Finding the Points of Intersection

To find the points of intersection between a line and a circle, one can substitute the equation of the line into the equation of the circle. This leads to a quadratic equation, which can be solved to determine the points of intersection.

Example Problem

Consider the circle defined by the equation:

$$ (x - 3)^2 + (y - 2)^2 = 16 $$

and the line defined by the equation:

$$ y = 2x - 1 $$

To find the points of intersection, we substitute the line's equation into the circle's equation:

1. Substitute y in the circle's equation:

$$ (x - 3)^2 + ((2x - 1) - 2)^2 = 16 $$
$$ (x - 3)^2 + (2x - 3)^2 = 16 $$

2. Expand and simplify:

$$ (x^2 - 6x + 9) + (4x^2 - 12x + 9) = 16 $$
$$ 5x^2 - 18x + 18 = 16 $$
$$ 5x^2 - 18x + 2 = 0 $$

3. Apply the quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$
where $a = 5$, $b = -18$, $c = 2$.

$$ x = \frac{18 \pm \sqrt{(-18)^2 - 4 \times 5 \times 2}}{2 \times 5} $$
$$ x = \frac{18 \pm \sqrt{324 - 40}}{10} $$
$$ x = \frac{18 \pm \sqrt{284}}{10} $$
$$ x = \frac{18 \pm 2\sqrt{71}}{10} $$
$$ x = \frac{9 \pm \sqrt{71}}{5} $$

4. Substitute x back into the line equation to find y:

$$ y = 2 \left(\frac{9 \pm \sqrt{71}}{5} \right) - 1 $$
$$ y = \frac{18 \pm 2\sqrt{71}}{5} - 1 $$
$$ y = \frac{18 \pm 2\sqrt{71}}{5} - \frac{5}{5} $$
$$ y = \frac{13 \pm 2\sqrt{71}}{5} $$

Thus, the points of intersection are:

$$ \left(\frac{9 + \sqrt{71}}{5}, \frac{13 + 2\sqrt{71}}{5} \right) $$
$$ \left(\frac{9 - \sqrt{71}}{5}, \frac{13 - 2\sqrt{71}}{5} \right) $$

Practice Problems

To solidify understanding, here are some practice problems related to lines that intersect circles. Try solving these problems independently before checking the answers.

1. Find the points of intersection between the circle $(x + 1)^2 + (y - 4)^2 = 25$ and the line $y = 3x + 2$.
2. Determine whether the line $x + 2y = 4$ is a secant, tangent, or does not intersect the circle $(x - 2)^2 + (y + 3)^2 = 9$.
3. For the circle defined by $(x - 4)^2 + (y + 1)^2 = 36$ and the line $y = -\frac{1}{2}x + 5$, find the points of intersection.

Answers to Practice Problems

1. Problem 1 Solution:
- Substitute $y = 3x + 2$ into $(x + 1)^2 + (3x + 2 - 4)^2 = 25$.
- Solve the resulting quadratic equation for x and substitute back to find y.

2. Problem 2 Solution:
- Check the distance from the center $(2, -3)$ to the line $x + 2y - 4 = 0$ using the formula for the distance from a point to a line.

- Compare the distance to the radius 3 to determine the type of intersection.

3. Problem 3 Solution:
- Substitute $y = -\frac{1}{2}x + 5$ into the circle's equation and solve for x.
- Use the quadratic formula to find the solutions and substitute back to find corresponding y values.

Conclusion

In conclusion, understanding how lines intersect with circles is a fundamental aspect of geometry that has practical applications in various fields, including engineering and physics. By mastering the techniques outlined in this article and practicing with additional problems, students can build confidence in their ability to tackle complex geometric challenges. Remember that the key to solving intersection problems lies in substituting the equations accurately and solving the resulting equations systematically.

Frequently Asked Questions

What are the key properties of lines that intersect circles?

Lines that intersect circles can either be secants, which cross the circle at two points, or tangents, which touch the circle at exactly one point. The angle formed by a tangent and a radius at the point of tangency is always 90 degrees.

How can I determine the point of intersection between a line and a circle?

To find the intersection points, you can substitute the equation of the line into the equation of the circle and solve for the coordinates. This will usually result in a quadratic equation that can be solved to find the intersection points.

What is the significance of the distance from the center of the circle to the line?

The distance from the center of the circle to the line helps determine the nature of the intersection. If the distance is less than the radius, the line intersects the circle at two points; if equal, it is tangent; if greater, there are no intersections.

Can a line intersect a circle at more than two points?

No, a straight line can intersect a circle at a maximum of two points. Any more intersections would violate the definition of a line, as it cannot curve.

What methods can be used to solve problems involving lines that intersect circles?

Common methods include using algebraic techniques to solve for intersection points, employing geometric properties to analyze relationships between angles and segments, and applying the Pythagorean theorem for distance calculations.

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